

# Thermodynamics and evaporation of the noncommutative black hole

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**ABSTRACT:** We investigate the thermodynamics of the noncommutative black hole whose static picture is similar to that of the nonsingular black hole known as the de Sitter-Schwarzschild black hole. It turns out that the final remnant of extremal black hole is a thermodynamically stable object. We describe the evaporation process of this black hole by using the noncommutativity-corrected Vaidya metric. It is found that there exists a close relationship between thermodynamic approach and evaporation process.

**KEYWORDS:** Black Holes, Non-Commutative Geometry, Models of Quantum Gravity.

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## 1. Introduction

Hawking's semiclassical analysis of the black hole radiation suggests that most information about initial states is shielded behind the event horizon and will not back to the asymptotic region far from the evaporating black hole [1]. This means that the unitarity is violated by an evaporating black hole. However, this conclusion has been debated by many authors for three decades [2–4]. It is closely related to the information loss paradox, which states the question of whether the formation and subsequent evaporation of a black hole is unitary. One of the most urgent problems in the black hole physics is the lack of resolution of the unitarity issue. Moreover, a complete description of the final stage of the black hole evaporation is important but is still quite unknown. In order to reach the solution to these problems, we have to use quantum gravity. Although two leading candidates for quantum gravity are the string theory and the loop quantum gravity, we need to introduce another approach that provides a manageable form of the quantum gravity effect. The holographic principle could serve such a purpose because it includes the effect of the quantum mechanics and gravity [5, 6].

Also it is interesting to consider the generalized uncertainty principle (GUP) since the Heisenberg uncertainty principle may not be satisfied when quantum gravitational effects become important [7–10]. We note that the GUP provides the minimal length scale and thus modifies the thermodynamics of a singular black hole at the Planck scale only.

On the other hand, even though the noncommutativity also provides the minimal length scale [7], this provides a totally different black hole: noncommutative black hole (NBH) [11]. This is similar to the nonsingular black hole with two horizons [12]. We think that it is very important to study the effects of noncommutativity on the terminal phase of black hole evaporation. In case of the Schwarzschild black hole, the temperature diverges and a large curvature state is reached. However, it was shown that the noncommutativity can cure this pathological short distance behavior [13].

In this work, we first study thermodynamic properties of the NBH thoroughly and then investigate its evaporation process. Especially, we wish to point out the connection between thermodynamic approach and evaporation process.

The organization of this work is as follows. We study the thermodynamics of the NBH in section II. Section III is devoted to investigation of the evaporation process of the NBH by introducing the noncommutativity-corrected Vaidya metric. Finally, we discuss and summarize our results in section IV.

## 2. Thermodynamics of noncommutative black hole

It was shown that noncommutativity eliminates point-like structures in favor of smeared objects in flat spacetime [11]. The effect of smearing is implemented by substituting the Dirac-delta function with Gaussian distribution of the width  $\sqrt{\theta}$ . This is a coordinate coherent approach to the noncommutativity with planck units of  $c = \hbar = G = 1$ . For this purpose, the mass density is chosen as

$$\rho_\theta(r) = \frac{M}{(4\pi\theta)^{\frac{3}{2}}} \exp\left(-\frac{r^2}{4\theta}\right), \quad (2.1)$$

which plays the role of a matter source, and the total mass  $M$  of the source is diffused throughout a region of linear size  $\sqrt{\theta}$ .  $\theta$  comes from the noncommutator of  $[x^\mu, x^\nu] = i\theta^{\mu\nu}$  with  $\theta^{\mu\nu} = \theta \text{diag}[\epsilon_1, \dots, \epsilon_{D/2}]$  [14–17]. We note that the constancy of  $\theta$  leads to a consistent treatment of Lorentz invariance and unitarity [18]. Eq. (2.1) provides a self-gravitating droplet of anisotropic fluid whose energy-momentum tensor is given by  $T^\mu{}_\nu = \text{diag}[-\rho_\theta, p_r, p_\perp, p_\perp]$  with the radial pressure  $p_r = -\rho_\theta$  and tangential pressure  $p_\perp = -\rho_\theta - \frac{1}{2}r\partial_r\rho_\theta$ . Solving the Einstein equation  $G_{\mu\nu} = 8\pi T_{\mu\nu}$  leads to the solution

$$ds^2 \equiv g_{\mu\nu}dx^\mu dx^\nu = -F(r)dt^2 + F(r)^{-1}dr^2 + r^2d\Omega_2^2. \quad (2.2)$$

Here the metric function  $F(r)$  is given by

$$F(r) = \left[1 - \frac{4M}{r\sqrt{\pi}}\gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right)\right], \quad (2.3)$$

where the lower incomplete gamma function is defined by

$$\gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right) \equiv \int_0^{\frac{r^2}{4\theta}} t^{\frac{1}{2}}e^{-t}dt. \quad (2.4)$$

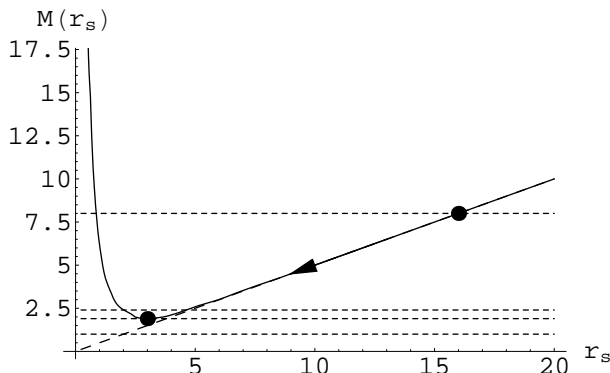
Note that when  $r$  goes to infinity,  $\gamma$  approaches to  $\sqrt{\pi}/2$ . From the condition of  $g_{00}(r_s) = 0$ , the event horizon can be found as

$$r_s = \frac{4M}{\sqrt{\pi}}\gamma\left(\frac{3}{2}, \frac{r_s^2}{4\theta}\right) \equiv \frac{4M}{\sqrt{\pi}}\gamma_s \quad (2.5)$$

which provides the mass  $M(r_s)$  as a function of the horizon radius  $r_s$

$$M(r_s) = \frac{\sqrt{\pi}r_s}{4\gamma_s}. \quad (2.6)$$

In the large radius of  $r_s^2/4\theta \gg 1$  (Hawking regime:  $r_s \simeq 2M$ ), the effect of noncommutativity can be neglected. On the other hand, at the short distance of  $r_s^2/4\theta \simeq \mathcal{O}(1)$  (critical



**Figure 1:** The solid line: mass  $M(r_s)$  as a function of the black hole radius  $r_s$ . The dashed line is the Schwarzschild case. Four horizontal lines are  $M = 8.0(= M_i)$ ,  $2.4(= M_m)$ ,  $1.9(= M_0)$ , and  $1.0(< M_0)$  from top to bottom. The mass  $M_i$  is introduced to be the initial mass ( $\bullet$ ) and  $M_0(\bullet)$  is the end point for an evaporation process ( $\rightarrow$ ). For  $M \geq M_0$ ,  $r_C(\leq r_0)$  describes the inner horizon, while  $r_s(\geq r_0)$  represents the outer horizon. de Sitter space appears for  $0 \leq r_s < r_C$ .

regime), one expects to find significant changes due to the spacetime noncommutativity. We note that although the metric in eq. (2.2) gives rise to asymptotically Schwarzschild spacetime, it is basically different from the Schwarzschild solution. This solution has two parameters  $M$  and  $\theta$ , in compared with one parameter  $M$  for the Schwarzschild case. Hereafter we choose  $\theta = 1$  for numerical computations without any loss of generality. As is shown figure 1, two masses of  $M(r_s)$  and  $M_{\text{Sch}} = r_s/2$  are different at the critical regime but the two are the same in the Hawking regime. A minimum mass of  $M(r_0) = M_0 = 1.9\sqrt{\theta}$  is determined from the condition of  $dM/dr_s = 0$ .

For definiteness, we consider three different types: 1) For  $M > M_0$  ( $M = M_i = 8.0\sqrt{\theta}$ ), two distinct horizons appear with the inner (Cauchy) horizon  $r_C$  and the outer (event) horizon  $r_s$  ( $r_C \leq r_0 \leq r_s$ ). 2) In case of  $M = M_0$ , one has the degenerate horizon at  $r_0 = 3.0\sqrt{\theta}$ , which corresponds to the extremal black hole. 3) For  $M < M_0$  ( $M = 1.0\sqrt{\theta}$ ), there is no horizon. In case of  $M \gg M_0$ , the inner horizon shrinks to zero, while the outer horizon approaches the Schwarzschild radius  $r_s = 2M$ . Hence the noncommutative black hole solution looks like the nonsingular solution known as the de Sitter-Schwarzschild black hole<sup>1</sup> [12]. Here  $\rho_\theta$  connects the de Sitter vacuum in the origin with the Minkowski vacuum at infinity.

The black hole temperature in the noncommutative geometry can be calculated to be

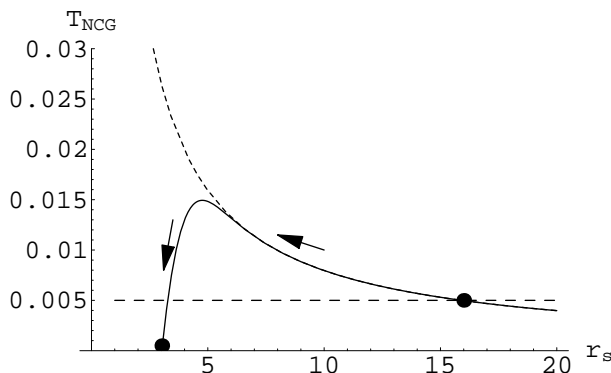
$$T_{NBH}(r_s) \equiv -\frac{1}{4\pi} \left[ \frac{dg_{00}}{dr} \right]_{r=r_s} = \frac{1}{4\pi r_s} \left[ 1 - \frac{r_s^3}{4\theta^{\frac{3}{2}}} \frac{e^{-\frac{r_s^2}{4\theta}}}{\gamma_s} \right]. \quad (2.7)$$

For  $r_s^2/4\theta \gg 1$ , one recovers the Hawking temperature of the Schwarzschild black hole

$$T_H = \frac{1}{4\pi r_s}. \quad (2.8)$$

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<sup>1</sup>At this point, one does not confuse this noncommutative black hole with the Schwarzschild-de Sitter black hole with  $r_s \leq r_C$ , which corresponds to a singular black hole inside the cosmological horizon [19].



**Figure 2:** The solid line: temperature  $T_{NBH}$  as a function of the black hole radius  $r_s$ . The dashed line is the Schwarzschild case. The horizontal dashed line denotes the initial temperature  $T_i = 0.005$ (●) and  $T = 0$ (●) represents the end point for an evaporation process ( $\rightarrow$ ).

Therefore, at the initial stage of Hawking radiation, the black hole temperature increases as the horizon radius decreases. It is important to investigate what happens as  $r_s \rightarrow \sqrt{\theta}$ . In the commutative case,  $T_H$  diverges and this puts limit on the validity of the conventional description of Hawking radiation. Against this scenario, the temperature  $T_{NBH}$  includes noncommutative effects, which are relevant at short distance comparable to  $\sqrt{\theta}$  [11]. As is shown in figure 2, the temperature of the NBH grows during its evaporation until it reaches to the maximum value  $T_{NBH} = T_m = 0.015$  at  $r_s = r_m = 4.76(M_m = 2.4)$  and then falls down to zero at  $r_s = r_0$  which the extremal black hole appears with  $T_{NBH} = 0$ . As a result, the noncommutativity restricts evaporation process to a planck-size remnant, similar to the GUP inspired black hole [9]. In the region of  $r < r_0$ , there is no black hole for  $M < M_0$  and thus the temperature can not be defined. For  $M > M_0$ , we have the inner horizon but the observer at infinity does not recognize the presence of this horizon. Hence we call this region as the forbidden region.

The entropy of the NBH can be obtained using the relation<sup>2</sup>

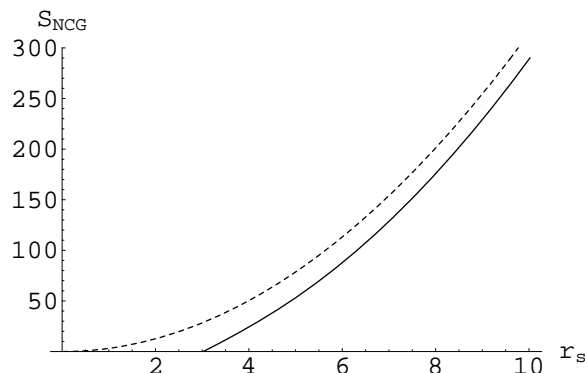
$$S_{NBH}(r_s) = \int_{M_0}^M \frac{dM'}{T_{NBH}(M')} = \int_{r_0}^{r_s} \frac{1}{T_{NBH}(r')} \left( \frac{dM'}{dr'} \right) dr'. \quad (2.9)$$

The numerical result of this integration is shown in figure 3. In this case we have zero entropy for the extremal black hole at  $r_s = r_0$ . On the other hand, we have the area-law behavior of  $S_{BH} = \pi r_s^2$  for the Schwarzschild black hole.

In order to check the thermal stability of the NBH, we have to know the heat capacity.

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<sup>2</sup>In deriving eq. (2.9), we use the first-law of thermodynamics  $dM = T_{NBH}dS_{NBH}$ . The issue is the lower bound of the integral. As was shown figure 1, we have two branches for  $M > M_0$ . The inner branch describes the inner cosmological horizon, while the outer branch shows the evolution of outer event horizon. We note that the first-law of thermodynamics holds for the outer horizon, because the observer at infinity does not recognize the inner horizon which is beyond the outer horizon. Furthermore, we have an unphysical negative temperature  $T_{NBH} < 0$  for  $r_s < r_0$  [20]. Hence the proper lower bound is not  $M = 0$  but  $M = M_0$ .



**Figure 3:** The solid line: entropy  $S_{NBH}$  as a function of the black hole radius  $r_s$ . The dashed line is the Schwarzschild case.

The heat capacity of the NBH is given by

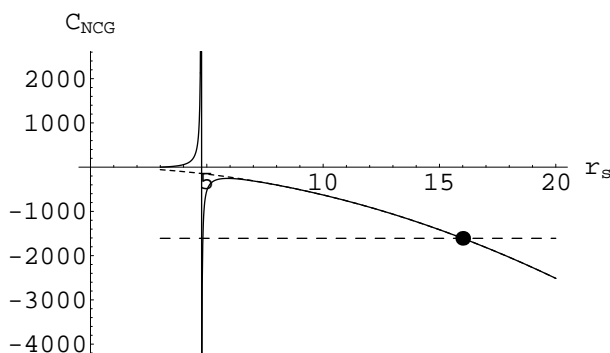
$$C_{NBH}(r_s) = \frac{dM}{dT_{NBH}} = \left( \frac{dM}{dr_s} \right) \left( \frac{dT_{NBH}}{dr_s} \right)^{-1} \quad (2.10)$$

and its variation is plotted in figure 4. Here we find a stable region of  $C_{NBH} > 0$  at the critical regime. This means that the NBH is thermodynamically stable in the range of  $r_0 < r_s < r_m$ . The heat capacity becomes singular at  $r_s = r_m$  which corresponds to the maximum temperature  $T_{NBH} = T_m$ . This picture is consistent with our expectation. At this stage, we raise a question: what extend is it physical that the thermodynamic process would pass through a point  $r_s = r_m$  where the specific heat goes from being infinitely negative to infinitely positive and then down to a finite positive? We may understand this picture from the analogy of the Hawking-Page phase transition in the AdS black hole [21, 22]. In the Hawking-Page transition, we start with the AdS space. A small black hole appears with negative heat capacity. The heat capacity changes from negatively infinity to positively infinity at the minimum temperature which corresponds to the maximum temperature of  $T_{NBH} = T_m(M = M_m, r_s = r_m)$  in our model. Then the large black hole with positive heat capacity comes out as a stable object. For our noncommutative black hole, we may consider the thermodynamic process as the inverse Hawking-Page transition. As a result, the infinite change at  $r_s = r_m$  indicates a thermodynamic behavior of the gravitating system which appears in the process from the large black hole to the extremal black hole. We also observe that a thermodynamically unstable region ( $C_{NBH} < 0$ ) appears for  $r_s > r_m$ . As a consistent check, we note that in the Hawking regime,  $C_{NBH}$  is consistent with the specific heat of the Schwarzschild black hole  $C_{Sch} = -2\pi r_s^2$ .

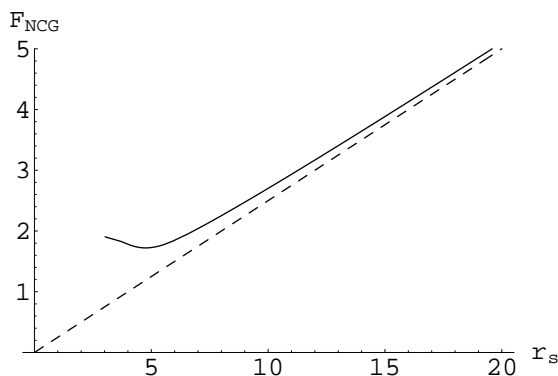
Finally, we introduce the free energy as

$$\begin{aligned} F_{NBH}(r_s) &= M(r_s) - T_{NBH}(r_s)S_{NBH}(r_s) \\ &= M(r_s) - \frac{1}{4\pi r_s} \left[ 1 - \frac{r_s^3}{4\theta^{\frac{3}{2}}} \frac{e^{-\frac{r_s^2}{4\theta}}}{\gamma_s} \right] \int_{r_0}^{r_s} \frac{1}{T_{NBH}} \frac{dM'}{dr'} dr'. \end{aligned} \quad (2.11)$$

Although we do not know its analytic form, its graph is shown in figure 5 by numerical computations. As expected, the free energy also has the minimum value at  $r_s = r_m = 4.76$ .



**Figure 4:** The solid line: heat capacity  $C_{NBH}$  as a function of the black hole radius  $r_s$ . The dashed curve is the Schwarzschild case. The horizontal dashed line denotes the initial heat capacity  $C_i = -1608(\bullet)$  for evaporation.



**Figure 5:** The solid line: plot of the free energy  $F_{NBH}$  as a function of  $r_s$ . The dashed line is the Schwarzschild case  $F_{Sch} = M/2 = r_s/4$ .

We find that two free energies take the same form in the Hawking regime. At the critical regime, two are different. We anticipate that the free energy  $F_{NBH}$  is negative at the critical regime because of positive heat capacity. However, this is positive.

At this stage we would like to compare our results with Nozari and Mehdipour [23]. They insisted that there exist negative temperature, negative entropy, and anomalous heat capacity in the thermodynamic study of the NBH. They argued that these unusual features show the failure of standard thermodynamics at the quantum gravity level. However, these results seem to be incorrect because they did not consider carefully the fact that the black hole prescription is meaningful only in the range of  $r_s = r_0(M = M_0)$  to  $\infty(M = \infty)$ . The observer at infinity talks about thermodynamics of the outer horizon. However, he does not know what happens inside the outer horizon. Our results are correct and consistent with those for the nonsingular black hole [24].

Finally we describe a thermodynamic process, which is closely related to the evaporation process of the NBH. Let us start with the black hole with mass  $M = M_i > M_0(r_i > r_0)$  in the Hawking regime. In this case we sketch the evaporation process ( $\rightarrow$ ) by observing

thermodynamic quantities: For a process of  $r_i \rightarrow r_m \rightarrow r_0$ , one has  $M_i = 8 \rightarrow M_m \rightarrow M_0$  (figure 1);  $T_i = 0.005 \rightarrow T_m \rightarrow T_0$  with a sequence of  $T_0 < T_i < T_m$  (figure 2). Interestingly, as is shown in figure 4, we have a change of  $C_i (= -1605) \rightarrow C_m (-\infty \rightarrow \infty) \rightarrow C_0 (= 0.0015)$ . Here we find that the final remnant of extremal black hole at  $r_s = r_0$  is thermodynamically stable because of positive heat capacity.

### 3. Evaporation of the noncommutative black hole

We start with the fact that the NBH looks like the regular solution known as the de Sitter-Schwarzschild black hole. Hence its causal structure is similar to that of a Reissner-Nordstrom black hole with the internal singularity replaced by a regular center. It is known that such a spacetime is unphysical because of the presence of the Cauchy horizon  $r_C$ . However, if the NBH evaporates, the Cauchy horizon is no more real than the event horizon  $r_s$ . Actually the evaporating process will terminate at the point which corresponds to the maximum Cauchy horizon and the minimum event horizon ( $r_C = r_0 = r_s$ ). Hence we do not need to worry about the presence of the Cauchy horizon. Also it is interesting to explore the evaporation process of the NBH because of the absence of a singularity. We guess that the dynamic regions are Vaidya-like with the negative-energy flux during evaporation.

We begin by reexpressing the metric in eq. (2.2) in terms of ingoing Eddington-Finkelstein coordinates:  $(v, r, \theta, \phi)$ . We introduce the advanced time coordinate

$$v = t + r^*, \quad r^* \equiv \int^r dr'/F(r'). \quad (3.1)$$

Here  $r^*$  is a generalization of the tortoise coordinate. Using  $dv = dt + dr/F(r)$ , we obtain

$$ds^2 = -F(r)dv^2 + 2dvdr + r^2d\Omega_2^2. \quad (3.2)$$

Considering the static metric together with Stefan's law, the mass dependence of the luminosity is given by

$$L(M) = \sigma AT_{NCG}^4 \quad (3.3)$$

with  $A = 4\pi r_s^2$  and  $\sigma = \pi^2/60$  for a single massless field with 2 degrees of freedom [25]. Now we are in a position to compute the mass  $M(v)$  of the black hole as seen by a distant observer at time  $v$  by solving the differential equation including first-order in the luminosity,

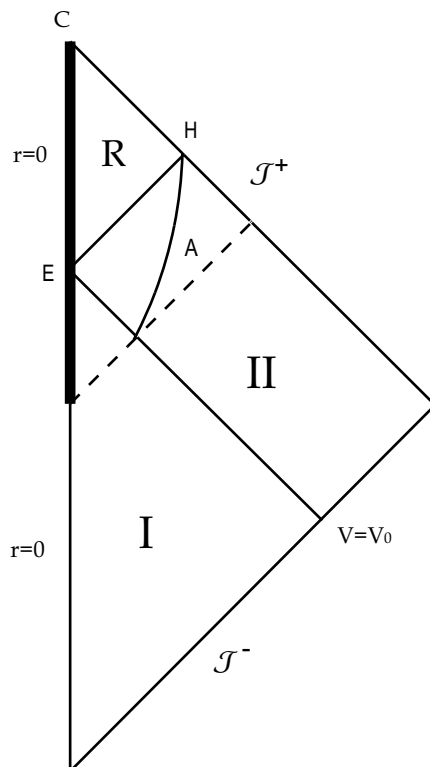
$$-\frac{d}{dv}M(v) = L(M(v)). \quad (3.4)$$

The improved metric is obtained by replacing the constant  $M$  in  $F(r)$  with  $M(v)$ :

$$ds_{NCV}^2 = -F(r, v)dv^2 + 2dvdr + r^2d\Omega^2, \quad F(r, v) = 1 - \frac{4M(v)\gamma}{r\sqrt{\pi}}. \quad (3.5)$$

For  $M \gg M_0$  ( $\gamma \simeq \sqrt{\pi}/2$ ), eq. (3.5) becomes the Vaidya metric, which was frequently used to explore the influence of the Hawking radiation on the Schwarzschild geometry [15, 26, 27]. It is a solution to Einstein equation  $G_{\mu\nu} = 8\pi T_{\mu\nu}$ , where  $T_{\mu\nu}$  describes an inward moving





**Figure 6:** The Penrose diagram of the evaporating NBH. Region *I* is a flat spacetime and region *II* is the evaporating NBH spacetime. Region *R* means a planck-size remnant. The line *EH* is the event horizon, the line *CH* is the Cauchy horizon, and the curve *A* is the apparent horizon. At  $V = V_0$ , an imploding null shell exists to balance the flux of negative energy.

null fluid. In this picture, the decreasing  $M$  is due to the inflow of negative energy. The metric in eq. (3.5) can be regarded as the noncommutativity-corrected Vaidya (NCV) metric.

It is instructive to ask which energy-momentum tensor  $T_{\mu\nu}$  would give rise to the NCV metric. Computing the Einstein tensor with eq. (3.5), one finds that non-zero components are

$$T^v_v = T^r_r = p_r, \tag{3.6}$$

$$T^r_v = \frac{\gamma \dot{M}(v)}{2(\pi)^{3/2} r^2}, \tag{3.7}$$

$$T^\theta_\theta = T^\phi_\phi = p_\perp. \tag{3.8}$$

Here the dot denotes the derivative with respect to  $v$ . Allowing for  $M(v) \neq \text{const}$ , the new feature is given by a nonzero component  $T^r_v$  which describes the inflow of negative energy into the black hole for  $\dot{M} < 0$ . This shows pure radiation, recovering the Vaidya solution for  $r^2/4 \gg 1$ . In the Vaidya case, the ingoing radiation creates a singularity. However, the center remains regular with de Sitter space. This implies that the noncommutative effects protect the core.

Even though  $F(r, v)$  is a complicated function of  $r$ , both the early and the late stages of the evaporation process can be described approximately. We assume that a black hole starts with  $M(v=0) > M_0$  in the Hawking regime. In this case, we have the known result of the Schwarzschild black hole

$$T_H(M) = \frac{1}{8\pi M}, \quad L_{\text{Sch}}(M) = \frac{\delta}{M^2} \quad (3.9)$$

with  $\delta = \frac{\sigma}{256\pi^3}$ . It is easy to solve the differential equation  $-\dot{M} = L(M)$  for this luminosity. With  $M(v=0) = M_i$  the solution takes the form

$$M(v) = \left[ M_i^3 - 3\delta v \right]^{1/3}. \quad (3.10)$$

This decreasing mass is valid during the early stage of the evaporation process, as long as  $M(v)$  is well above the minimal mass  $M_0$ . If one extrapolates (3.10) to small mass, one finds  $M(v_*) = 0$ . This implies that a final explosion with  $T \rightarrow \infty$  and  $L \rightarrow \infty$  occurs, after a finite time of  $v_* = M_i^3/(3\delta)$ . However, this is not possible for the NBH. The final stage of the evaporation process, where the cold remnant forms, is at the critical regime. It can be described by those terms which are dominant for  $M \rightarrow M_0$ . Using the Taylor's expansion of eq. (2.7) at  $M_0$  together with eq. (3.3), we obtain the approximate forms:

$$T_{\text{NBH}}(M) \simeq \alpha(M - M_0), \quad (3.11)$$

$$L_{\text{NBH}}(M) \simeq \beta(M - M_0)^4. \quad (3.12)$$

with  $\alpha = dT_{\text{NBH}}/dM|_{r_s=r_0} = 3242.87$  and  $\beta = \sigma A\alpha^4 = 2.1 \times 10^{15}$ . Solving  $-\dot{M} = L(M)$  with eq. (3.12), one finds

$$M(v) - M_0 = \frac{M_1 - M_0}{[1 + 3\beta(M_1 - M_0)^3(v - v_1)]^{1/3}} \quad (3.13)$$

where  $v_1$  is a time in the critical region and  $M_1 = M(v_1)$ . For  $v \rightarrow \infty$ , the difference<sup>3</sup> of  $M(v) - M_0$  vanishes as  $v^{-1/3}$ . Hence, we have the late stage of evaporation:  $T_{\text{NBH}}(v) \propto v^{-1/3}$  and  $L_{\text{NBH}} \propto v^{-4/3}$ . The noncommutativity-corrected black hole spacetime leads to concrete predictions on the final state of the evaporation process. We note again that  $M = M_0$  is the mass of a cold remnant, which is an extremal black hole with the planck size [28].

Finally, the whole picture of evaporation process is shown in figure 6. Region *I* is a flat spacetime, while at  $V = V_0$  an imploding null shell is present.<sup>4</sup> Strictly speaking, it must have a negative tension in order to balance the flux of negative energy on its future side [29]. Region *II* corresponds to the evaporating NBH spacetime. The apparent horizon *A* is a timelike hypersurface which meets the event horizon at future null infinity in the Penrose diagram. The null ray of dashed line, which is tangent to the earliest portion of the apparent horizon, would have been the event horizon if the black hole were not radiating. The final remnant *R* of the NBH is an extremal black hole whose inner and outer horizons have the same radius of  $r_s = r_C = r_0$ .

<sup>3</sup>In the case of quantum-corrected Newton's constant, it was shown that the difference of  $(M(v) - M_0)$  vanishes as  $v^{-1}$  by using the RG improved Vaidya metric [25].

<sup>4</sup> $V$  is the Kruskal advanced time coordinate, defined as  $V = -e^{-\kappa v}$  with  $\kappa = 2\pi T_{\text{NBH}}$  the surface gravity of the outer horizon.

## 4. Discussion and summary

There are a lot of approaches for treating black hole evaporation process. This process is a quantum gravitational effects and its understanding provides a suitable framework toward a complete formulation of quantum gravity. In this work, we have focused on the thermodynamic approach to the final stage of NBH evaporation. Our results and corresponding figures indicate stable features when the mass of the NBH becomes planck scale. We have obtained a maximum temperature  $T_{NBH} = T_m$  that the NBH can reach before cooling down to absolute zero ( $T_{NBH} = T_0$ ). Moreover, the entropy is zero at  $M = M_0$  and the heat capacity is positive at the critical region. These imply that the final remnant is a thermodynamically stable object. The thermodynamic process is closely connected to the evaporation process using the NCV metric. In this case the backreaction effect is trivial because the temperature approaches zero (not divergent) as  $M \rightarrow M_0$ .

Finally, we have shown that the coordinate coherent approach to the noncommutativity can cure the singularity problem at the final stage of black hole evaporation.

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